Integer Wavelet Transform and Predictive Coding Technique for Lossless Medical Image Compression

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Abstract—Lossless image compression has one of its important application in the field of medical images. Enormous amount of data is created by the information present in the medical images either in multidimensional or multiresolution form. Efficient storage, transmission, management and retrieval of the voluminous data produced by the medical images has nowadays become increasing complex. Solution to the complex problem lies in the lossless compression of the medical data. Medical data is compressed in such a way so that the diagnostics capabilities are not compromised or no medical information is lost. This paper proposes a hybrid technique for lossless medical image compression that combines integer wavelet transforms and predictive coding to enhance the performance of lossless compression. Here we will first apply the integer wavelet transform and then predictive coding to each subband of the image obtained as an output to lifting scheme. Measures such as entropy, scaled entropy and compression ratio are used to evaluate the performance of the proposed technique.

Keywords—Lossless Compression; Integer Wavelet Transform; Predictive Coding; Medical Image, entropy.

I. INTRODUCTION

Recent developments in health care practices and development of distributed collaborative platforms for medical diagnosis have resulted in the development of efficient technique to compress medical data. Telemedicine applications involve image transmission within and among health care organizations using public networks. In addition to compressing the data, this requires handling of security issues when dealing with sensitive medical information systems for storage, retrieval and distribution of medical data. Some of the requirements for compression of medical data include high compression ratio and the ability to decode the compressed data at various resolutions.

In order to provide a reliable and efficient means for storing and managing medical data computer based archiving systems such as Picture Archiving and Communication Systems (PACS) and Digital-Imaging and Communications in Medicine (DICOM) standards were developed. Health Level Seven (HL7) standards are widely used for exchange of textual information in healthcare information systems. With the explosion in the number of images acquired for diagnostic purposes, the importance of compression has become invaluable in developing standards for maintaining and protecting medical images and health records.

There has been a lot of research going on in lossless data compression. The most common lossless compression algorithms are run-length encoding, LZW, DEFLATE, JPEG, JPEG 2000, JPEG-LS, LOCO-I etc. Lempel–Ziv–Welch is a lossless data compression algorithm which can be used to compress images. The performance of LZW can be enhanced by introducing three methods. The first two methods eliminate the frequent flushing of dictionary, thus lowering processing time and the third method improves the compression ratio by reducing number of bits transferred over the communication channel. JPEG is mostly commonly used lossy compression technique for photographic images which can be converted into lossless by performing integer reversible transform. Lossless compression in JPEG [7] is achieved by performing integer reversible DCT (RDCT) instead of the floating point DCT used in original JPEG on each block of the image later using lossless quantization. Lossless JPEG does not allow flexibility of the code stream, to overcome this JPEG 2000[1-2] has been proposed. This technique performs lossless compression based on an integer wavelet filter called biorthogonal 3/5. JPEG 2000’s lossless mode runs really slow and often has less compression ratios on artificial and compound images. To overcome this drawback JPEG-LS [6] has been proposed. This is a simple and efficient baseline algorithm containing two distinct stages called modeling and encoding. This technique is a standard evolved after successive refinements as shown in articles [3], [4], and [5]. JPEG-LS algorithm is more scalable than JPEG and JPEG 2000.

II. LOSSLESS IMAGE COMPRESSION MODEL

Many image compression algorithms use some form of transform coding. Fig. 1 shows a block diagram of encoder and
decoder using transform coding. The first step is to obtain a mathematical transformation to the image pixels in order to reduce the correlation between the pixels. The result of the transform is known as the transform coefficients. After this step, in lossy compression, an explicit quantizer may be used, or an implicit quantizer such as the truncation of the bitstream may be used. The source of the data loss in image compression is the quantizer. Thus, in the lossless compression case, the quantizer is not used. The third step is coefficient coding, which means that the transform coefficients are reorganized in order to exploit properties of the transform coefficients and obtain new symbols to be encoded at the fourth step. For example, the transform coefficients can be considered as a collection of quad-trees or zero-trees [8] [9] and or treated in a bit plane fashion, so as to provide scalability to the compressed bitstream. The symbols from the coefficient coding are losslessly compressed at the entropy coding step. Entropy coding can be any method capable of compressing a sequence of symbols, such as Huffman coding [10], arithmetic coding [11] and Golomb coding [12].

![Block Diagram of an Encoder and Decoder using Transform Coding](image)

**III. LOSSLESS COMPRESSION CODING TECHNIQUES**

In this section different coding techniques used to achieve lossless compression are discussed. The primary encoding algorithms used to produce bit sequences are entropy coding techniques of which the most efficient are Huffman coding (also used by DEFLATE) and arithmetic coding. We also go over lossless predictive coding technique.

**A. Entropy Coding**

Entropy measures the amount of information present in the data or the degree of randomness of the data. After the data has been quantized into a finite set of values it can be encoded using an entropy coder to achieve additional compression using probabilities of occurrence of data. This technique reduces the statistical redundancy. The entropy coder encodes the given set of symbols with the minimum number of bits required to represent them. It is a variable length coding which means that it assigns different number of bits to different gray levels. If the probability of occurrence is more, then fewer bits/sample will be assigned.

**ENTROPY (H):** Suppose we have M input levels or symbols (S1, S2...SM) with their probabilities (P1, P2..., PM)

$$ H = - \sum_{k=1}^{M} P_k \log_2 P_k = \sum_{k=1}^{M} P_k \log_2 \left( \frac{1}{P_k} \right) $$

In the least random case it takes only one value where

$$ H = 0 $$

Most random case:

$$ H = \log_2 M $$

The average number of bits per pixel needed with Huffman coding is given by

$$ R = \sum_{k=1}^{M} P_k N_k $$

Where Pk represent the probabilities of the symbols and Nk represent the number of bits per the code generated. Coding efficiency ($\eta$) can also be calculated using $H$ and $R$ generated earlier

$$ \eta = \frac{H}{R} \times 100 $$

**B. Huffman Coding**

Huffman coding is an entropy coding algorithm which is used in lossless compression. In this technique the two smallest probabilities are combined or added to form a new set of probabilities. This uses a variable length code table which is based on the estimated probability of occurrence for each possible value of the source symbol. This is developed by David, A. Huffman. In Huffman coding each symbol is represented in a specific method which expresses the most common characters with fewer strings than used for any other character. Huffman coding is equivalent to simple binary block encoding. Although Hoffmanns original algorithm is optimal for a symbol-by-symbol coding (i.e. a stream of unrelated symbols) with a known input probability distribution. It is not optimal when the symbol-by-symbol restriction is dropped, or when the probability mass functions are unknown, not identically distributed, or not independent.

The basic technique involves creating a binary tree of nodes which can be finally stored as an array. This size depends on the number of symbols which have given probabilities. Now the lowest two probabilities will be added and one probability will be represented by „0” and the other probability which is added will be assigned a „1”. This process is repeated until all the additions are completed leaving a sum of one. The simplest construction algorithm uses a priority queue where the node with lowest probability is given highest priority. The performance of the method is calculated using entropy .

**IV. INTEGER WAVELET TRANSFORM**

The wavelet transform generally produces floating-point coefficients. Although the original pixels can be reconstructed by perfect reconstruction filters without any loss in principle, the use of finite-precision arithmetic and quantization prevents perfect reconstruction. The reversible IWT (Integer Wavelet Transform), which maps integer pixels to integer coefficients and can reconstruct the original pixels without any loss, can be used for lossless compression [13] [14] [15] [16]. One approach used to construct the IWT is the use of the lifting scheme (LS) described by Calderbank et al. The IWT construction using lifting is done in the spatial domain, contrary to the frequency domain implementation of a traditional wavelet transform [16] [17].

Wavelet transforms have proven extremely effective for transform-based image compression. Since many of the wavelet transform coefficients for a typical image tend to be very small or zero, these coefficients can be easily coded. Thus, wavelet transforms are a useful tool for image compression.
The main advantage of wavelet transforms over other more traditional decomposition methods (like the DFT and DCT) is that the basis functions associated with a wavelet decomposition typically have both long and short support. The basis functions with long support are effective for representing slow variations in an image while the basis functions with short support can efficiently represent sharp transitions (i.e., edges). This makes wavelets ideal for representing signals having mostly low-frequency content mixed with a relatively small number of sharp transitions. With more traditional transforms techniques like the DFT and DCT, the basis functions have support over the entire image, making it difficult to represent both slow variations and edges efficiently.

V. LIFTING SCHEME

The simplest lifting scheme is the lazy wavelet transform, where the input signal is first split into even and odd indexed samples.

\[(\text{odd}_{j-1} , \text{even}_{j-1}) = \text{Split}(s_j)\]

The samples are correlated, so it is possible to predict odd samples from even samples which in the case of Haar transform are even values themselves. The difference between the actual odd samples and the prediction becomes the wavelet coefficients. The operation of obtaining the differences from the prediction is called the lifting step. The update step follows the prediction step, where the even values are updated from the input even samples and the updated odd samples. They become the scaling coefficients which will be passed on to the next stage of transform. This is the second lifting step.

\[d_{j-1} = \text{odd}_{j-1} - P(\text{even}_{j-1})\]
\[s_{j-1} = \text{even}_{j-1} + U(d_{j-1})\]

Finally the odd elements are replaced by the difference and the even elements by the averages. The computations in the lifting scheme are done in place which saves lot of memory and computation time. The lifting scheme provides integer coefficients and so it is exactly reversible. The total number of coefficients before and after the transform remains the same.

\[
\begin{align*}
\text{Even}_{j-1} & = s_{j-1} - U(d_{j-1}) \\
\text{Odd}_{j-1} & = d_{j-1} + P(\text{Even}_{j-1}) \\
\text{Finally} & = s_j = \text{Merge} (\text{Even}_{j-1}, \text{Odd}_{j-1})
\end{align*}
\]

The inverse transform gets back the original signal by exactly reversing the operations of the forward transform with a merge operation in place of a split operation. The number of samples in the input signal must be a power of two, and these samples are reduced by half in each succeeding step until the last step which produces one sample.

The transformed image in Fig.5 shows different sub bands of which the first sub band is called LL which represents the low resolution version of the image, the second sub band is called LH which represents the horizontal fluctuations, the third band is called HL which represents the vertical fluctuations, and the fourth sub band is called the HH which represents the diagonal fluctuations. Same procedure can be followed to obtain different levels of image decomposition by changing the inputs given to the lifting or filter bank implementation techniques.

VI. INTRODUCTION TO PREDICTIVE CODING

The prediction technique computes the weighted differences between neighboring pixel values to estimate the predicted pixel value. The prediction error is decomposed by a one-level integer wavelet transform to improve the prediction. The differences are taken between the original sample and the
sample(s) before the original sample. Let \( f(n) \) be the original sample then the difference \( d(n) \) will be given by

\[
d(n) = f(n) - f(n-1).
\]

Figure 6. Original Histogram

Figure 7. Histogram of the difference

Fig.7 shows that it is easier to encode the difference rather than encoding the original sample because of less dynamic range.

\[
\hat{f}(n) = \langle f(n-1) \rangle
\]

Figure 8. Predictive Encoder

Generally, the second order predictor is used which is also called Finite Impulse Response (FIR) filter. The simplest predictor is the previous value, in this experiment the predicted value is sum of the previous two values with alpha and beta being the predictor coefficients.

\[
\hat{f}(n) = \alpha \cdot f(n-1) + \beta \cdot f(n-2)
\]

\( \hat{f}(n) \) is the rounded output of the predictor, \( f(n-1) \) and \( f(n-2) \) are the previous values, \( \alpha \) and \( \beta \) are the coefficients of the second order predictor ranging from 0 to 1. The output of the predictor is rounded and is subtracted from the original input. This difference is given by

\[
d(n) = f(n) - \hat{f}(n)
\]

Now this difference is given as an input to the decoder part of the predictive coding technique. In the decoding part the difference is added with the \( f^*(n) \) to give the original data

\[
f(n) = d(n) + \hat{f}(n)
\]

VII. IMPLEMENTATION AND EXPERIMENTAL RESULTS

In this report the Integer Wavelet Transform (IWT) and the Predictive Coding Techniques are used to perform lossless image compression. The performance of the proposed techniques is calculated by finding the Entropy and scaled entropy of the compressed image. The performance is also measured using compression ratio which is given by the ratio of the bits in the original uncompressed data to the number of bits in the compressed data.

A. Procedure

The procedure of the implementation involves two methods of performing compression on the medical image. In the first method IWT is performed first followed by predictive coding. The procedure of the implementation involves two methods of performing compression on the medical image. In the first method IWT is performed first followed by predictive coding technique on the transformed image. The second method involves reduction of the filter coefficients by a factor of 3/2 and then applying integer wavelet transform followed by predictive coding technique. All these methods use Haar filter in the lifting scheme and the filter coefficients are given by:

\[
h1 = [-1 9 9 1]/(16);
\]

\[
h2 = [0 0 1 1]/(-4);
\]

Where \( h1 \) are the prediction filter coefficients and \( h2 \) are the update filter coefficients in the lifting scheme.

The reduced filter coefficients are given by

\[
h1 = [-1 9 9 1]/(16*1.5);
\]

\[
h2 = [0 0 1 1]/(-4*1.5);
\]

B. Implementation using Method1
In this method integer wavelet transform is applied on the image which divides the image into four subbands ss, sd, ds, dd. Now predictive coding is applied on the four different bands separately giving outputs d1, d2, d3 and d4. The reconstruction process involves applying the predictive decoding followed by inverse integer transform. The reconstructed image is represented by z. To verify the perfect reconstruction the original and the reconstructed images are subtracted and the output is a dark image with maximum and minimum values as zero.

C. Outputs of Method1

D. Implementation using Method2

In this method the filter coefficients used in the integer wavelet transform using lifting scheme are reduced by a factor of 3/2 and the same steps mentioned in Section VII-B are performed.

E. Outputs of Method2

IV. CONCLUSION

This paper presented two different methods for lossless medical image compression and these methods are tested using four different medical images of 256x256. The images are compressed losslessly by performing integer wavelet transform using lifting technique as mentioned in the report of Daubechies and Wim Sweldens and lossless predictive coding technique using second order predictors. Lifting is achieved by...
performing simple filtering steps using finite filters such as Haar filter. In all our methods we have used first order Haar filter for performing lifting. In lossless predictive coding technique we take the difference or prediction error into consideration rather than taking into account the original sample/image, the differences are taken between the original sample and the sample(s) before the original sample as it is easier to encode the difference rather than encoding the original sample.

In the first lossless compression method, the image is transformed into four subbands using lifting technique, then predictive coding is applied to each subband using different predictor coefficients alpha and beta, giving an encoded image as output. Entropy and scaled entropy are used to calculate the performance of the system, which calculates the number of bits per pixel. A lower entropy and scaled entropy indicate higher performance of the system.

The analysis of the experimental results has given many conclusions. Choosing the predictor coefficients is more critical as the alpha and beta value can lie between 0 and 1, so different combinations of these coefficients are tested. The best combination in methods 1 and 2 are (0.01, 0.01) & (0.9, 0.9) respectively has been highlighted in the table 1 & 2. Compression ratio for different media images have been calculated and have been listed in table 5. Among the two methods, the second method of performing integer wavelet transform followed by predictive coding using the reduced filter coefficients gave a better compression. Fig. 23 (See Appendix) shows the comparison of minimum entropy for method 1 and method 2 using different values of alpha and beta. Out of all the medical images used the rmi of ankle give the minimum entropy using method 1 and using method 2 the nasal fracture gave the least entropy. These results can be seen from the graphs plotted in Fig. 24 (See Appendix).

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REFERENCES

APPENDIX

TABLE I. TABULATIONS OF METHOD 1 USING DIFFERENT VALUES OF ALPHA AND BETA

<table>
<thead>
<tr>
<th>S.NO.</th>
<th>ALPHA</th>
<th>BETA</th>
<th>ORIGINAL ENTROPY</th>
<th>ENTROPY AFTER IWT</th>
<th>ENTROPY AFTER PREDICTIVE CODING</th>
<th>SCALED ENTROPY</th>
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</thead>
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<td>0.1</td>
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### TABLE II. Tabulations of Method 2 using different values of alpha and beta

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<th>ORIGINAL ENTROPY</th>
<th>ENTROPY AFTER IWT</th>
<th>ENTROPY AFTER PREDICTIVE CODING</th>
<th>SCALED ENTROPY</th>
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### TABLE III. Comparison Table for different images using Method 1

<table>
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<th>IMAGE</th>
<th>ORIGINAL ENTROPY</th>
<th>ENTROPY AFTER INTEGER WAVELET TRANSFORM</th>
<th>ENTROPY AFTER PREDICTIVE CODING</th>
<th>SCALED ENTROPY</th>
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<td>MRI OF BRAIN</td>
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<td>0.8773</td>
<td>0.9871</td>
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### TABLE IV. Comparison Table for different images using Method 2

<table>
<thead>
<tr>
<th>IMAGE</th>
<th>ORIGINAL ENTROPY</th>
<th>ENTROPY AFTER INTEGER WAVELET TRANSFORM</th>
<th>ENTROPY AFTER PREDICTIVE CODING</th>
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<tr>
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### GRAPHS

![Comparison of Entropy's of method 1 and method 2 using different alpha and beta](image)

Figure 23. Graph between predictors and scaled entropy for method 1 and method 2
Figure 24. Comparing the behavior of different medical images (256x256)

TABLE V. COMPRESSION RATIO OF DIFFERENT MEDICAL IMAGES

<table>
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